

# Fragmentation and Brittleness: Richard Stacey

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**Mathematics In Industry Study Group - 2016**

January 15, 2016

# Introduction

- ▶ Tough rocks produce larger fragments after blasting. There is a standard model for size estimation.
- ▶ Recently a law case arose whereby the standard model didn't correctly determine fragment size.
- ▶ Tarasov has defined a novel brittleness index for rocks based on experimental lab tests.

Question: Can this be used to improve estimates for fragment size and distribution?

## The Standard Model: Kuz-Ram model

$$x_m = AK^{-0.8}Q^{1/6}\left(\frac{115}{RWS}\right)^{\frac{19}{20}}$$

where:

$x_m$ : Mean particle size.

K: Powder factor (kg explosive/ $m^3$ )

Q: mass of explosive in hole (kg)

A: a rock 'factor' (0.8-22 !)

RWS: The relative weight strength of the explosive used.

This formula doesn't take into account features of the blast (rock type, bore hole spacing, geometry of the site....)

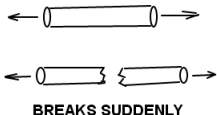
$$R_x = \exp[-0.693(x/x_m)^n] \text{ with } n = 0.7 - 2$$

Note especially that there is no term in the equation that explicitly takes into account rock properties except A. (eg .brittleness).

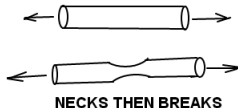
# Stress Strain Curves: Tarasov (Brittleness Index)

## Definition of the Brittle and Ductile rock

### BRITTLE MATERIAL



### DUCTILE MATERIAL

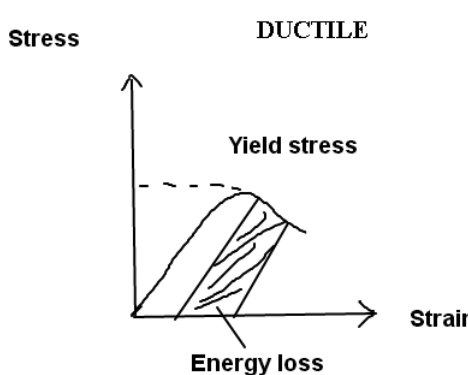
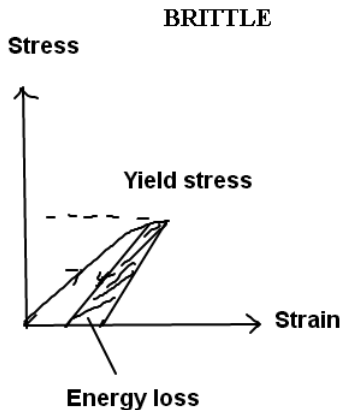


- ▶ Brittle rocks crack.
- ▶ Ductile rocks 'neck'.
- ▶ In general rocks are neither one nor the other.
- ▶ Brittleness is used to describe one of the rock characteristics.

Energy loss is greater for a ductile rock.

## Stress Strain Curves: Tarasov (Brittleness Index)

**The brittleness 'index' ( $\beta$ ):** Defines the extent to which a rock is 'Brittle/Hard' (Class 2) or 'Ductile' (Class 1) using a stress/strain lab test.



Note the very different shape after the yield stress is exceeded.  
The brittleness index  $\beta$  is the area ratio (shaded/total)

**Question: Why does this matter?**

The Tarasov index quantifies the **energy loss** associated with stress application!

For a hard/brittle rock much more elastic energy is retained after rupture which means little energy goes into 'Cracking'.

**Can this index be used to obtain a better result!**

# Possible Models

- ▶ Energy/Scaling/Statistical model.
- ▶ Two mechanistic models.
- ▶ Composite models.

## A Scaling/Energy Model: Mean particle size

The aim is to improve on the Kuz-Ram model using dimensional analysis.

### Model parameters

- ▶ **Yield stress**  $Y = \frac{M}{T^2 L}$ .
- ▶ **Brittleness** index ( $\beta$ ) is dimensionless.
- ▶ **Energy per unit time per unit volume** due to explosive charge  
 $= \epsilon = \frac{M}{T^3 L}$ .
- ▶ Energy available for fracturing per unit time and volume  $= \beta \epsilon$ .
- ▶ **Speed of propagation of elastic wave** (primary wave)  $= C_P = \frac{L}{T}$
- ▶ Mean fragment size  $= X_m = L$
- ▶ A is a universal constant: **applies to all material**.

**We combine the above parameters to obtain a dimensionally consistent expression for fragment size.**

The possible combinations are:

$$X_m = AY^a(\beta\epsilon)^b C_P^c = A\left(\frac{M}{T^2 L}\right)^a \left(\beta \frac{M}{LT^3}\right)^b \left(\frac{L}{T}\right)^c$$



# Dimensional analysis: Mean particle size

Dimensionally compatible providing:

- ▶  $L : 1 = -a - b + c$
- ▶  $M : 0 = a + b$
- ▶  $T : 0 = -2a - 3b - c$

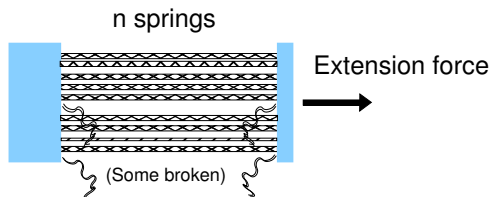
By solving the last equation we will find that  $a = 1, b = -1, c = 1$ .  
This gives:

$$***x_m = A \frac{Y C_P}{\beta \epsilon} ***$$

Where  $C_P = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$   $E$  is Young's modulus,  $\nu$  Poisson's Ratio,  $\rho$  the density.

Note that this formula includes the important rock properties and is the only combination that makes dimensional sense!

# Breaking Springs Model - Simple 1d approach



- ▶ Springs ( $n_0$ ) are stretched by an external force  $\mathcal{T}_{ext}$ .
- ▶ Individual springs have same spring constant ( $k$ )
- ▶ .... but have different breaking strengths  $\mathcal{T}_s^{crit}$ .
- ▶ External force increased  $\Rightarrow$  some springs break
- ▶ ..... remaining ( $n$ ) springs bear the load.

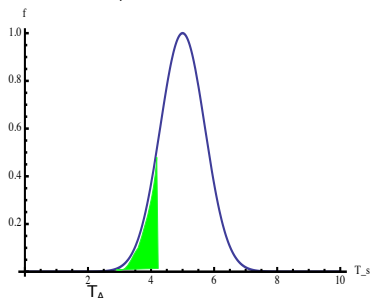
Can model mimic experimental stress-strain results (Class 1 & 2)?

If "Yes", correlate equivalent parameters.

# Spring Breakage Distribution

Intact rock corresponds to intact springs, cracks correspond to broken springs. If we assume a normal distribution for breakage:

$$f = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left[\frac{(T_s - \bar{T}_s)^2}{2\sigma^2}\right]}$$



The green shading shows springs that have broken after the application of an individual spring tension  $T_s$ .

# Equations

Single spring: Tension  $T_s$  causes displacement  $x$  (initial length  $l_0$ ):

$$T_s = kx$$

Multiply by number of intact springs  $n \Rightarrow$  stress/strain reln:

$$nT_s \equiv \mathcal{T}_{\text{ext}} = (nkl_0) \frac{x}{l_0} \equiv E_{\text{eff}} \frac{x}{l_0}$$

The effective Young's modulus is defined in terms of  $k$  and  $n$ .

$$E_{\text{eff}} = E_0 \left( \frac{n}{n_0} \right)$$

# Springs

Distribution gives number of survivors supporting the load:

$$\frac{n}{n_0} = 1 - \int_0^{T_s} f(T_s) dT_s$$

$$\text{so } E_{eff} = E_0 \left[ 1 - \int_0^{T_s} f(T_s) dT_s \right]$$

For Normal distribution (the exact result):

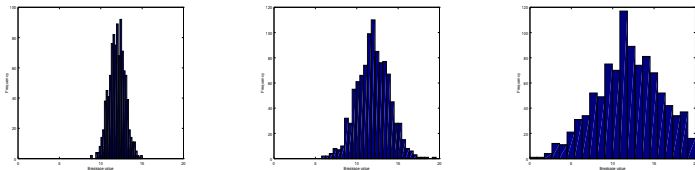
$$E_{eff}(T_s) = E_0 \left[ 1 - \text{Erf} \left( \frac{T_s - \bar{T}_s}{\sqrt{2\pi}\sigma} \right) \right]$$

and the stress/strain results can be obtained.

$$\mathcal{T}_{ext} = nT_s = E_0 \left[ 1 - \text{Erf} \left( \frac{T_s - \bar{T}_s}{\sqrt{2\pi}\sigma} \right) \right] \frac{x}{l_0} \quad (1)$$

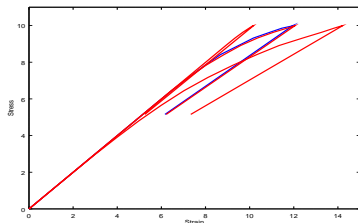
.... unfortunately, don't know  $n$ ,  $T_s$ .

# 1d model Simulations - 1



Randomly generated normal dsns. of spring breakage tensions for  $\sigma = 1, 2, 4$ . Left to right - decreasing brittleness.  $N = 1000$  springs/bonds.

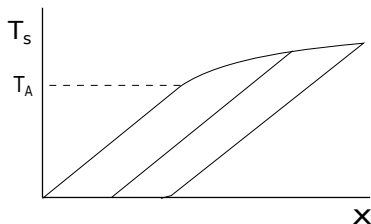
## 1d model Simulations - 2



- ▶ Corresponding stress-strain relation diagrams. Less brittle  $\Rightarrow$  wider curve.
- ▶ Agrees well with Class 2 materials
- ▶ Distribution of bond breakage/cracking should correlate to particle sizes

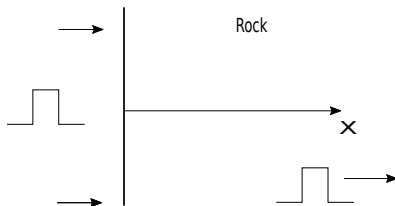
# Results

- ▶ The results asymptote to a  $T_s$  with all springs broken
- ▶ If applied stress is cycled there is offset, but process repeats
- ▶ The rate of approach to the asymptote depends on the distribution width  $\sigma$ .
- ▶ One can associate rock characteristics with model parameters (Young's modulus, Yield strength, brittleness)
- ▶ The shape is right for Class 2 brittle rocks.  
But Class 1 models aren't covered;  $k$  variations needed?



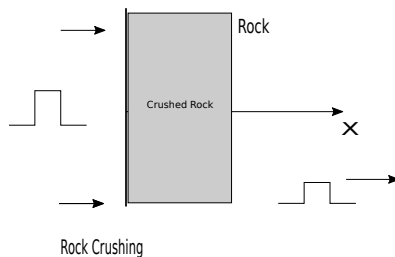


# A Continuum State Change Model



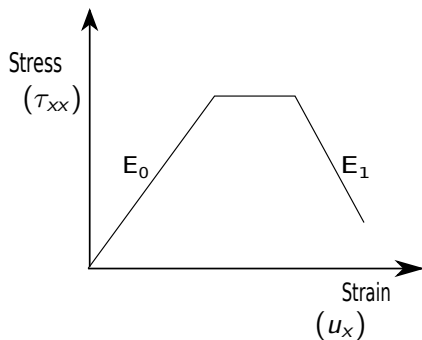
- ▶ The end of a semi-infinite rock face (or rod) is impulsively hit.
- ▶ If stress levels generated are less than fracture levels  $T_{crit}$  then a longitudinal pressure pulse travels away from the face at speed  $\sqrt{E_0/\rho}$ .
- ▶ If stress levels exceed  $T_{crit}$  then the rock will partially crush/crack.

# Rock Crushing



Note that the transmitted stress wave is reduced due to rock crushing.

# Equations



fig

Newton's Law gives:  $\tau_{xx,x} = \rho u_{tt}$ , so that with  $\tau_{xx} = E^* u_x$ , we get

$$E^* u_{xx} = \rho u_{tt}$$

in the (damaged, damaged) and undamaged regions resp.

# Ductile and Brittle Rocks

- ▶ Note that across the front the equation changes from elliptic to wave type for ductile materials, but not for brittle materials. **Interesting!**
- ▶ Thus in the brittle case the energy decays slowly and the wave travels a great distance. For ductile materials the impulse is quickly damped.
- ▶ The extent of damage (cracking) can be assessed using a state change idea. The internal energy of the cracked rock is different.
- ▶ The primary aim of the analysis is to determine the speed of travel of the front, the extent of propagation, and the expected fragment size.

## Conclusions: The Models

- ▶ The energy model. If it works then it could be really important. Needs checking with data
- ▶ Springs model: early days but is promising.
- ▶ State change model: needs development.